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THE $A \cdot B \cdot A^{-1}$ PRODUCT IN THE LIGHT OF ABBREVIATED MATRIX SYMBOLS

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A b s t r a c t. If $B = n_2/M_2N_2P_2/ = b_{ij}$ /rotation angle β / and $A = n_1/M_1N_1P_1/ = a_{ij}$ /rotation angle $\propto$ / the product $A \cdot B \cdot A^{-1}$ can be written:

$$n_1/M_1N_1P_1/ \cdot n_2/M_2N_2P_2/ \cdot n_1/\bar{M}_1\bar{N}_1\bar{P}_1/ = n_2/M_xN_xP_x/$$

where

$$\left\| \begin{matrix} M_x \\ N_x \\ P_x \end{matrix} \right\| = a_{ij} \cdot \left\| \begin{matrix} M_2 \\ N_2 \\ P_2 \end{matrix} \right\|$$

The angle $\mathcal{E}$ between the axes $n_2/M_2N_2P_2/$ and $n_2/M_xN_xP_x/$ fulfills the equation

$$\cos \mathcal{E} = R_1 \cdot E_{12}^2 + \cos \propto$$

INTRODUCTION

The $A \cdot B \cdot A^{-1}$ matrix product appears very often in theoretical treatment of conjugate symmetry operations. Let us consider two matrices a_{ij} and b_{ij} and calculate the conjugated matrix c_{ij}

$$a_{ij} \cdot b_{ij} \cdot a_{ij}^{-1} = c_{ij}$$

$$\left\| \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix} \right\| \times \left\| \begin{matrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{matrix} \right\| \times \left\| \begin{matrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{matrix} \right\| = \left\| \begin{matrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{matrix} \right\|$$

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Direct multiplication gives for c_{11} the following expression:

$$c_{11} = b_{11}\cdot a_{11}^2 + b_{12}\cdot a_{11}\cdot a_{12} + b_{13}\cdot a_{11}\cdot a_{13} + b_{21}\cdot a_{11}\cdot a_{12} + b_{22}\cdot a_{12}^2 + b_{23}\cdot a_{12}\cdot a_{13} + b_{31}\cdot a_{11}\cdot a_{13} + b_{32}\cdot a_{12}\cdot a_{13} + b_{33}\cdot a_{13}^2$$

Application of the generalized matrix

Introducing for b_{ij} the abbreviated symbol $n_2/M_2N_2P_2/$ where

$$\frac{360}{n_2} = \beta$$

and making use of the generalized matrix we can write

$$c_{11} = /M_2^2R_2 + \cos\beta\ /\cdot a_{11}^2 + /N_2^2R_2 + \cos\beta\ /\cdot a_{12}^2 + /P_2^2R_2 + \cos\beta\ /\cdot a_{13}^2 + 2\cdot M_2N_2R_2\cdot a_{11}\cdot a_{12} + 2\cdot M_2P_2R_2\cdot a_{11}\cdot a_{13} + 2\cdot N_2P_2R_2\cdot a_{12}\cdot a_{13}$$

and finally

$$c_{11} = /a_{11}\cdot M_2 + a_{12}\cdot N_2 + a_{13}\cdot P_2/^2\cdot R_2 + \cos\beta$$

Comparing this equation with the c_{11}-term of the generalized matrix we can state that the resulting matrix c_{ij} is constructed in the following way: the matrix a_{ij} acts on the $M_2\ N_2\ P_2$ values transforming them into new $M_xN_xP_x$ - values

$$M_x = a_{11}\cdot M_2 + a_{12}\cdot N_2 + a_{13}\cdot P_2$$

$$N_x = a_{21}\cdot M_2 + a_{22}\cdot N_2 + a_{23}\cdot P_2$$

$$P_x = a_{31}\cdot M_2 + a_{32}\cdot N_2 + a_{33}\cdot P_2$$

The matrix a_{ij} transforms the $M_2N_2P_2$ only, the rotation angle β remains thereby unchanged.

THE ANGLE ε BETWEEN CONJUGATED MATRIX AXES

The angle ε between $M_2N_2P_2$ and $M_xN_xP_x$ fulfills the equation

$$\cos\varepsilon = M_2M_x + N_2N_x + P_2P_x$$

Introducing for $M_x\ N_x\ P_x$ the values calculated above we obtain

$$\cos\varepsilon = R_1/M_1M_2 + N_1N_2 + P_1P_2/^2 + \cos\alpha$$

In these calculations the abbreviated symbol $n_1/M_1N_1P_1/$ with rotation angle α has been introduced for a_{ij}.

R E F E R E N C E S

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Józef NEDOMA

ILOCZYN $A\cdot B\cdot A^{-1}$ W ŚWIETLE SKRÓCONYCH SYMBOLI MACIERZOWYCH

S t r e s z c z e n i e

Jeżeli $B = n_2/M_2N_2P_2/ = b_{ij}$ /kąt obrotu β / oraz $A = n_1/M_1N_1P_1/ = a_{ij}$ /kąt obrotu α / wówczas iloczyn $A\cdot B\cdot A^{-1}$ można napisać w postaci

$$n_1/M_1N_1P_1/\cdot n_2/M_2N_2P_2/\cdot n_1/\bar{M}_1\bar{N}_1\bar{P}_1/ = n_2/M_xN_xP_x/$$

gdzie

$$\begin{Vmatrix} M_x \\ N_x \\ P_x \end{Vmatrix} = a_{ij}\cdot\begin{Vmatrix} M_2 \\ N_2 \\ P_2 \end{Vmatrix}$$

Kąt ε pomiędzy osiami $n_2/M_2N_2P_2/$ i $n_2/M_xN_xP_x/$ spełnia równanie

$$\cos\varepsilon = R_1\cdot E_{12}^2 + \cos\alpha$$

Иосиф НЭДОМА

ПРОИЗВЕДЕНИЕ $A\cdot B\cdot A^{-1}$ В СВЕТЕ СОКРАЩЕННЫХ МАТРИЧНЫХ СИМВОЛОВ

Р е з ю м е

Если $B = n_2/M_2N_2P_2/ = b_{ij}$ /угол вращения $-\beta$ / и $A = n_1/M_1N_1P_1/ = a_{ij}$ /угол вращения $-\alpha$/ тогда произведение $A\cdot B\cdot A^{-1}$ можно записать

$$n_1/M_1N_1P_1/\cdot n_2/M_2N_2P_2/\cdot n_1/\bar{M}_1\bar{N}_1\bar{P}_1/ = n_2/M_xN_xP_x/$$

$$\begin{Vmatrix} M_x \\ N_x \\ P_x \end{Vmatrix} = a_{ij}\cdot\begin{Vmatrix} M_2 \\ N_2 \\ P_2 \end{Vmatrix}$$

Угол ε между направлениями осей $n_2/M_2N_2P_2/$ и $n_2/M_xN_xP_x/$ удовлетворяет уравнению

$$\cos \varepsilon = R_1 \cdot E_{12}^2 + \cos \alpha$$